PLACE®

# STUDY GUIDE

**04 Mathematics**


Program for Licensing Assessments for Colorado Educators®

CO-SG-FLD004-03

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# PART 1: GENERAL INFORMATION ABOUT THE PLACE® AND TEST PREPARATION

Part 1 of this study guide is contained in a separate PDF file. Click the link below to view or print this section:

General Information About the PLACE and Test Preparation

# PART 2: FIELD-SPECIFIC INFORMATION

## TEST FIELD 04: MATHEMATICS

## INTRODUCTION

This section includes a list of the test objectives, immediately followed by a set of practice multiple-choice questions. For test areas that include a performance assessment (Basic Skills, all languages other than English, Special Education Specialist: Visually Impaired), one or more practice performance assignments (as applicable) will also be included.

***TEST OBJECTIVES.*** As noted earlier, the test objectives are broad, conceptual statements that reflect the knowledge, skills, and understanding an entry-level educator needs to teach effectively in a Colorado classroom. The list of test objectives represents the **only** source of information about what a specific test will cover.

***PRACTICE MULTIPLE-CHOICE QUESTIONS.*** The practice multiple-choice questions included in this section are designed to give you an introduction to the nature of the questions included on the PLACE test. The practice questions represent the various types of multiple-choice questions you may expect to see on an actual test; however, they are **not** designed to provide diagnostic information to help you identify specific areas of individual strengths and weaknesses or to predict your performance on the test as a whole.

When you answer the practice multiple-choice questions, you may wish to use the answer key to check your answers. To help you identify how the test objectives are measured, the objective statement to which each multiple-choice question corresponds is listed in the answer key. When you are finished with the practice questions, you may wish to go back and review the entire list of test objectives and descriptive statements for your test area.

***CALCULATORS.*** For the Mathematics (04) test, you must bring your own graphing calculator but you may not bring a calculator manual. Graphing calculators will not be provided at the administration. Only the brands and models listed below may be used. Approved calculator brands and models are subject to change; if there is a change, you will be notified. Test administration staff will clear the memory of your calculator both before and after testing. Therefore, be sure to back up the memory on your calculator, including applications, to an external device before arriving at the test site.

| Manufacturer | Approved Models |
|---|---|
| Casio | FX 1.0 PLUS, FX-7400G, FX-7400G PLUS, FX-9750G PLUS, CFX-9850G, CFX-9850G PLUS, CFX-9850Ga, CFX-9850Ga PLUS, CFX-9850GB PLUS, CFX-9850GB PLUS(WE), CFX-9850GC PLUS, CFX-9970G, ALGEBRA FX 2.0 (ALGFX2.0) |
| Sharp | EL-9300, EL-9600, EL-9600c, EL-9900 |
| Texas Instruments | TI-73, TI-80, TI-81, TI-82, TI-83, TI-83 Plus, TI-83 Plus Silver, TI-84, TI-84 Plus, TI-84 Plus Silver, TI-85, TI-86, TI-89, TI-89 Titanium |
| Hewlett-Packard | HP 9g, HP 40g, HP 49g, HP 49g PLUS |

# TEST FIELD 04: MATHEMATICS

Foundations of Mathematics
Functions and Relations
Measurement and Geometry
Probability and Statistics
Calculus and Discrete Mathematics

## FOUNDATIONS OF MATHEMATICS

**Understand number relationships and problem-solving strategies.**

Includes:

- using equivalent forms of a number (e.g., fraction, decimal, percent, scientific notation) to solve problems
- using ratios and proportions to solve problems
- analyzing and applying a variety of numerical problem-solving strategies (e.g., invented and standard algorithms, estimation, mental arithmetic)
- applying technology in problem-solving situations (e.g., graphing calculator)

**Understand the structure and properties of the real and complex number systems and their subsystems.**

Includes:

- analyzing similarities and differences among subsets of the real numbers (e.g., the rational numbers versus the real numbers)
- applying properties of the real numbers to arithmetic and algebraic operations
- solving problems involving multiple representations of the real numbers (e.g., roots, powers, infinite decimal expansions)
- using and performing operations on multiple representations of the complex numbers (e.g., algebraic, geometric, trigonometric, exponential)

**Understand the principles of and connections among number theory and linear and abstract algebra.**

Includes:

- applying concepts of prime numbers and divisibility (e.g., prime factorization, greatest common factor, least common multiple, modular arithmetic)
- analyzing and applying the properties and algebra of vectors and matrices to solve problems (e.g., applications to systems of equations, the geometry of matrix transformations, invertibility of matrices)
- identifying the basic properties of groups, rings, and fields

**Understand concepts of mathematical reasoning, communication, and the history of mathematics.**

Includes:

- translating among algebraic, geometric, numeric, graphic, and written modes of representing mathematical ideas
- converting between standard English language and mathematical language (e.g., notation, symbols)
- demonstrating knowledge of the language of mathematical statements and proof (e.g., contrapositive, proof by contradiction)
- assessing the validity of a mathematical argument
- recognizing the development and use of mathematics throughout history and across cultures

## FUNCTIONS AND RELATIONS

**Understand the properties of functions.**

Includes:

- evaluating functions (e.g., f(2), f($x + h$))
- analyzing the properties of functions (e.g., domain, range, even/odd)
- determining the effects of transformations (e.g., f($x + k$), $kf(x)$) on the graphs of functions
- analyzing operations on functions (e.g., inverse, composition)
- connecting a variety of representations of functions (e.g., algebraic, geometric, graphic, tabular, numeric, verbal)

**Understand linear functions.**

Includes:

- analyzing the relationships among algebraic, geometric, graphic, tabular, and verbal representations of linear functions
- interpreting properties of linear equations in context (e.g., slope, intercepts)
- solving and analyzing systems of linear equations and inequalities using a variety of methods (e.g., analytical, graphical)
- applying linear equations and inequalities to model real-world phenomena and solve problems in other disciplines

**Understand quadratic functions.**

Includes:

- finding and analyzing real and complex roots of quadratic functions
- analyzing the relationships among algebraic, geometric, graphic, tabular, and verbal representations of quadratic functions
- solving and analyzing quadratic equations and systems of equations using a variety of methods (e.g., analytical, graphical)
- applying quadratic equations and inequalities to model real-world phenomena and solve problems in other disciplines

**Understand polynomial, rational, absolute-value, and radical functions.**

Includes:

- finding and analyzing the real and complex roots of polynomial functions
- analyzing the relationships among algebraic, geometric, graphic, tabular, and verbal representations of polynomial, rational, absolute-value, and radical functions
- solving and analyzing polynomial, rational, absolute-value, and radical equations using a variety of methods (e.g., analytical, graphical)
- applying polynomial, rational, absolute-value, and radical functions to model real-world phenomena and solve problems in other disciplines

**Understand exponential, logarithmic, and trigonometric functions.**

Includes:

- applying the laws of exponents and logarithms to solve problems
- recognizing the inverse relationship between exponential and logarithmic functions and converting between the two forms
- analyzing connections among right triangle trigonometry, the unit circle, and trigonometric functions
- analyzing the relationships among algebraic, geometric, graphic, tabular, and verbal representations of exponential, logarithmic, and trigonometric functions
- solving and analyzing exponential, logarithmic, and trigonometric equations using a variety of methods (e.g., analytical, graphical)
- applying exponential, logarithmic, and trigonometric functions to model real-world phenomena and solve problems in other disciplines

## MEASUREMENT AND GEOMETRY

**Understand concepts, systems, and units of measurement.**

Includes:

- solving problems involving length, area, volume, capacity, time, temperature, angle, weight, and mass
- analyzing precision and error in measurement (e.g., percentage error, rounding error)
- converting within and between systems of measurement
- applying methods of indirect measurement (e.g., similarity, triangulation)

**Understand trigonometry and Euclidean geometry in two and three dimensions.**

Includes:

- using properties of points, lines, planes, and geometric figures to solve problems
- applying the Pythagorean theorem and right triangle trigonometry
- using similarity and congruence relationships
- analyzing connections between two- and three-dimensional figures (e.g., nets, projections, cross sections)

**Understand connections between algebra and geometry.**

Includes:

- representing figures in two- and three-dimensional coordinate systems
- analyzing translation, rotation, and reflection in the coordinate plane
- applying slope, distance, and midpoint formulas to investigate figures
- recognizing connections between algebra and geometry (e.g., conic sections, area representations of algebraic operations)

**Understand the axiomatic structure of geometry.**

Includes:

- identifying underlying principles of an axiomatic system
- recognizing and applying deductive and inductive reasoning
- making, testing, justifying, and proving conjectures and geometric constructions
- recognizing properties and examples of non-Euclidean geometries

## PROBABILITY AND STATISTICS

**Understand the theory of probability.**

Includes:

- determining empirical probabilities through analysis of data
- modeling and solving problems using principles (e.g., dependence, independence, conditionality) and techniques (e.g., tree diagrams, combinatorics, area models) of theoretical probability
- recognizing connections between theoretical and empirical probability
- applying binomial, uniform, and normal distributions to problem-solving situations

**Understand descriptive statistics.**

Includes:

- evaluating real-world situations to determine appropriate data-collection techniques
- constructing tables, charts, and graphs (e.g., box plots, histograms, scatter plots) to represent statistical data
- interpreting statistical data represented in tables, charts, and graphs (e.g., box plots, histograms, scatter plots)
- evaluating arguments and drawing conclusions based on a set of data

**Understand methods of data analysis.**

Includes:

- finding and interpreting percentiles, mean, median, mode, range, standard deviation, and variance
- recognizing the effects of data transformations on measures of central tendency and variability
- drawing conclusions about distributions of data based on statistical summaries
- applying technology (e.g., graphing calculator) to analyze a set of data
- using correlation to analyze the relationship between two sets of data

**Understand inferential statistics.**

Includes:

- analyzing the effect of sampling techniques and sample size on the results of a statistical study (e.g., survey, experiment)
- interpreting confidence intervals
- formulating and testing hypotheses

## CALCULUS AND DISCRETE MATHEMATICS

**Understand limits, continuity, and average rates of change.**

Includes:

- investigating the limits of functions, infinite sequences, and series
- analyzing the asymptotic behavior of functions
- interpreting continuity and discontinuity geometrically and analytically
- interpreting average rate of change as the slope of a secant line

**Understand differentiation and its application to problem-solving situations.**

Includes:

- recognizing the derivative of a function as the limit of the slope of a secant line (e.g., the difference quotient)
- interpreting the derivative in context and as an instantaneous rate of change
- using first and second derivatives to investigate the behavior of a function
- using differentiation to analyze and solve problems

**Understand integration and its application to problem-solving situations.**

Includes:

- using algebraic and geometric techniques to approximate the area under a curve
- interpreting the definite integral as the area under a curve
- applying the fundamental theorem of calculus to analyze connections between integration and differentiation
- using integration to model and solve real-world problems

**Understand the fundamental principles of discrete mathematics.**

Includes:

- using recursive formulas and difference equations
- modeling problems using graph theory
- solving problems involving permutations and combinations
- demonstrating an understanding of proof by mathematical induction
- analyzing sequences and series and using them to model and solve problems

## SAMPLE MATHEMATICS FORMULAS

| Formula | Description |
|---|---|
| $V = \frac{1}{3}Bh$ | Volume of a right cone and a pyramid |
| $A = 4\pi r^2$ | Surface area of a sphere |
| $V = \frac{4}{3}\pi r^3$ | Volume of a sphere |
| $A = \pi r\sqrt{r^2 + h^2}$ | Lateral surface area of a right circular cone |
| $S_n = \frac{n}{2}[2a + (n-1)d] = n\left(\frac{a + a_n}{2}\right)$ | Sum of an arithmetic series |
| $S_n = \frac{a(1 - r^n)}{1 - r}$ | Sum of a geometric series |
| $\sum_{n=0}^{\infty} ar^n = \frac{a}{1 - r},\ \lvert r\rvert < 1$ | Sum of an infinite geometric series |
| $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ | Distance formula |
| $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ | Midpoint formula |
| $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$ | Slope |
| $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ | Law of sines |
| $c^2 = a^2 + b^2 - 2ab\cos C$ | Law of cosines |
| $s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n - 1}$ | Variance |
| $s = r\theta$ | Arc length |
| $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ | Quadratic formula |

**Calculators for the Mathematics Test (Test Field 04)**

You must bring your own graphing calculator to the administration. Only the brands and models listed in the current PLACE Registration Bulletin may be used.

The approved calculator brands and models are subject to change. If there is a change, examinees will be notified.

**Administration staff will clear the memory of your calculator both before and after the test.** Be sure you back up the memory on your calculator, including applications, to an external device before arriving at the test site.

1. Use the expression below to answer the question that follows.

$$\frac{1}{a + bi} + \frac{1}{a - bi}$$

Which of the following is equivalent to the expression above where $i$ is the imaginary unit?

A. $\frac{2a}{a^2 + b^2}$

B. $\frac{2a}{a^2 - b^2}$

C. $\frac{1}{a^2 + b^2}$

D. $\frac{1}{a}$

2. If today is Thursday, what day of the week will it be 1000 days from today?

A. Sunday

B. Monday

C. Tuesday

D. Wednesday

3. Use the formula below to answer the question that follows.

$$F = G\frac{m_1 m_2}{r^2}$$

The formula above states Newton's law of gravitation where $F$ represents the gravitational force, $G$ is a universal constant, $m_1$ and $m_2$ represent the masses of 2 objects, and $r$ represents the distance between the 2 objects. Which of the following statements describes this law?

A. The gravitational force is directly proportional to the product of the masses of the 2 objects and inversely proportional to the square of the distance between the 2 objects.

B. The gravitational force varies linearly with the masses of the 2 objects and quadratically with the distance between the 2 objects.

C. The gravitational force is inversely proportional to the product of the masses of the 2 objects and directly proportional to the square of the distance between the 2 objects.

D. The gravitational force varies linearly with the masses of the 2 objects and exponentially with the distance between the 2 objects.

4. Given that $f(x) = 1 - x$, which of the following is equivalent to $2f(3x + 1)$?

A. $8 - 6x$

B. $-6x$

C. $4 - 6x$

D. $2 + 4x - 6x^2$

5. Use the table below to answer the question that follows.

| $x$ | f($x$) |
|---|---|
| 0 | $K$ |
| −4 | 0 |
| −6 | 0 |

If f($x$) is a function of the form f($x$) = $ax^2 + bx + c$, then which of the following expressions represents the coefficient $a$?

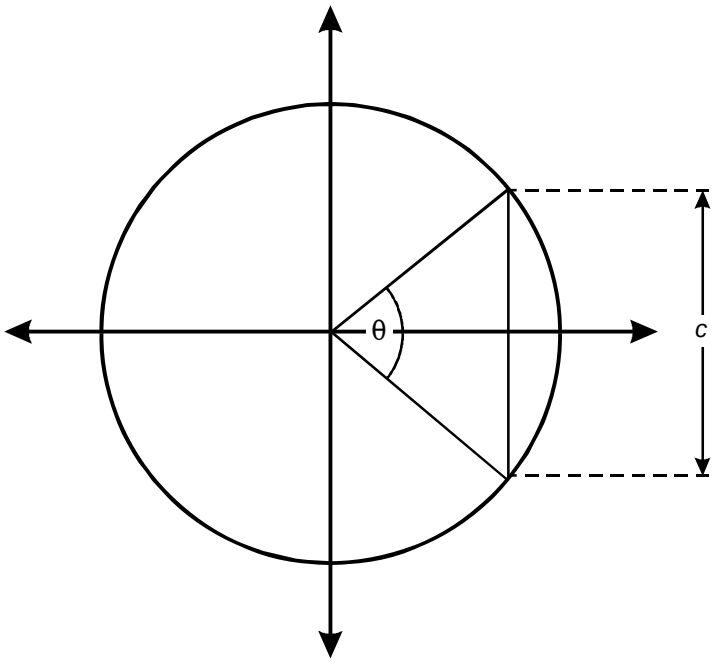
A. $\frac{10}{K}$

B. $\frac{K}{10}$

C. $\frac{24}{K}$

D. $\frac{K}{24}$

6. Use the diagram below to answer the question that follows.




A chord of length $c$ subtends central angle $\theta$ on a unit circle. Which of the following equations represents the relationship between $\theta$ and $c$?

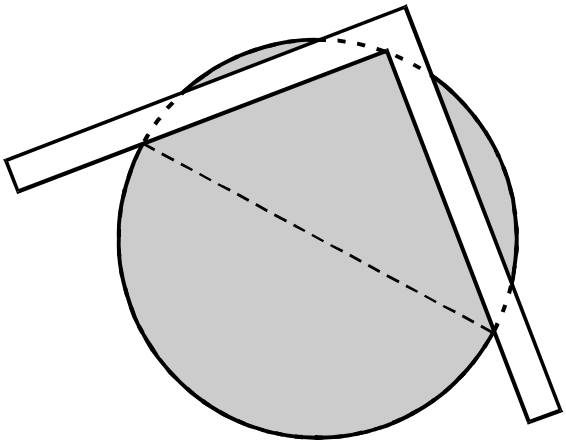
A. $c = \sin \theta$

B. $c = \sec \theta$

C. $c = 2 \sin\left(\frac{\theta}{2}\right)$

D. $c = 2 \sec\left(\frac{\theta}{2}\right)$

7. Use the diagram below to answer the question that follows.



A carpenter uses a large metal square that forms a right angle as shown above to find and mark a diameter line on a circular tabletop. Which of the following theorems justifies this measurement technique?

A. The measure of an inscribed angle is equal to half the measure of the inscribed arc.

B. The measure of an angle formed by a chord and a tangent is equal to half the measure of the intercepted arc.

C. A diameter that is perpendicular to a chord bisects the chord and its arc.

D. In the same circle or in congruent circles, 2 minor arcs are congruent if and only if their central angles are congruent.

8. $\triangle PQR$ has coordinates $P(0,0)$, $Q(2a,0)$, and $R(2a,4a)$. If $\triangle PQR$ is rotated 90° counterclockwise about the origin to form $\triangle P'Q'R'$, what are the coordinates of point $R$?

A. $(-2a, -4a)$

B. $(-2a, 4a)$

C. $(-4a, -2a)$

D. $(-4a, 2a)$

9. A coin is tossed 10 times, yielding 4 heads. The same coin is tossed 1000 times, yielding 492 heads. Which of the following statements best explains these results?

   A. The theoretical probability is an upper bound for the empirical probability.

   B. The longer the coin is tossed, the more reliable the theoretical probability becomes at predicting the next outcome.

   C. As the number of trials increases, the empirical probability becomes arbitrarily close to the theoretical probability.

   D. Theoretical probability is based on mathematical models, and hence it can never be as exact as empirical probability.

10. A national restaurant chain provides self-addressed comment cards on the tables of their restaurants in a sampling of major cities. Which of the following changes would most effectively make the information gained from this survey more representative of all customers' experiences?

    A. Discard data from the smaller cities involved in the survey.

    B. Administer the survey over the Internet instead of on paper.

    C. Provide an incentive for customers to fill out the cards.

    D. Provide comment cards only to customers who request them.

11. What is the average rate of change of the function $F(x) = 2x^2 + 3$ over the interval $x = -1$ to $x = 2$?

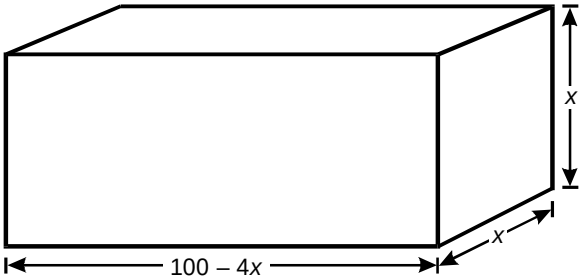
A. $-4$

B. $-2$

C. 2

D. 4

12. Use the diagram below to answer the question that follows.




The diagram shows the dimensional restrictions for a shipping box. Which of the following equations should be solved to find the value of $x$ that maximizes the volume of the box?

A. $100 - 8x = 0$

B. $200x - 12x^2 = 0$

C. $100x^2 - 4x^3 = 0$

D. $33.3x^3 - x^4 = 0$

# ANSWER KEY: MATHEMATICS

| Question Number | Correct Response | Objective |
|---|---|---|
| 1. | **A** | Understand the structure and properties of the real and complex number systems and their subsystems. |
| 2. | **D** | Understand the principles of and connections among number theory and linear and abstract algebra. |
| 3. | **A** | Understand concepts of mathematical reasoning, communication, and the history of mathematics. |
| 4. | **B** | Understand the properties of functions. |
| 5. | **D** | Understand quadratic functions. |
| 6. | **C** | Understand exponential, logarithmic, and trigonometric functions. |
| 7. | **A** | Understand concepts, systems, and units of measurement. |
| 8. | **D** | Understand connections between algebra and geometry. |
| 9. | **C** | Understand the theory of probability. |
| 10. | **C** | Understand inferential statistics. |
| 11. | **C** | Understand limits, continuity, and average rates of change. |
| 12. | **B** | Understand differentiation and its application to problem-solving situations. |